Module 7
Solving Complex Problems

Lesson 1 The Towers of Hanoi

Students will solve the Towers of Hanoi Puzzle. Students will learn how adding a new disk to the puzzle will significantly increase the amount of time required to solve the puzzle.

Computer Internet Projector CD Resource

Lesson 2 The Travelling Salesman Problem

Students will create solutions to the Travelling Salesman Problem. Each time a new city is added to the journey of a travelling salesman the amount of possible routes that can be travelled increases at an exponential rate.

Computer Internet Projector CD Resource

Lesson 1 – The Towers of Hanoi

Resources:
Towers of Hanoi (Resource 1), Towers of Hanoi in Scratch (Resource 2)

Key Vocabulary:
Towers of Hanoi, Exponential Growth

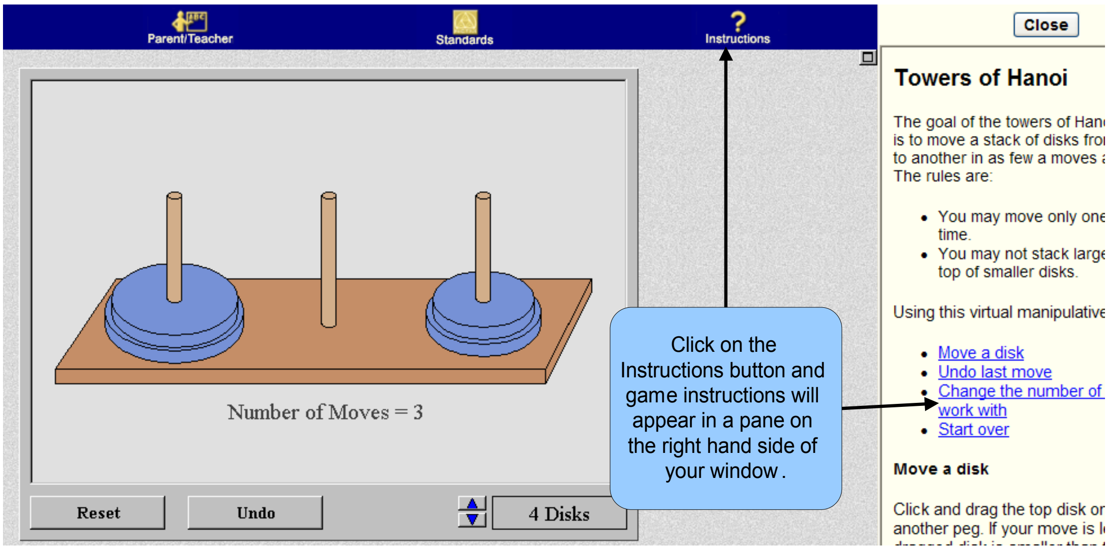
Description:
Students will investigate how the amount of time required to execute an algorithm increases as the size of the input changes. Students will solve the Towers of Hanoi puzzle for 3 disks. They should be able to solve the puzzle in 7 moves. They should be able to solve the puzzle for 4 disks in 15 moves. In general you can complete the puzzle in (2^n -1) moves, where n is the number of disks. By following this pattern students will notice how the number of moves increases significantly as more disks are added.

Learning Objectives:
1. To discover how to solve the Towers of Hanoi problem.
2. To investigate the increase in amount of time required to solve the Towers of Hanoi puzzle in relation to the increase in the number of disks used.

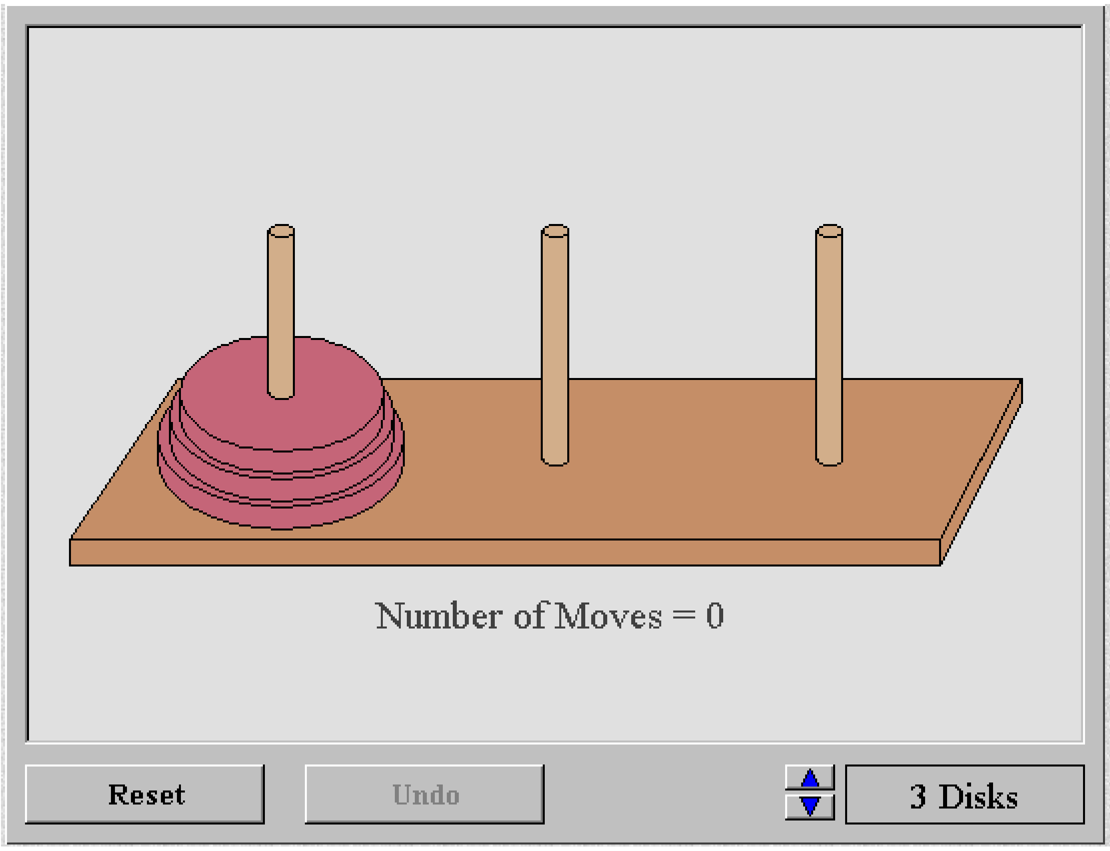
Lesson Introduction:
- Open the PowerPoint Presentation for the lesson Towers of Hanoi (Resource 1). Present slides 2 and 3.
- Introduce the mowing the lawn problem to demonstrate linear complexity.
- Tell students that a problem that increases in complexity very quickly each time a new input is added will be investigated.

Lesson Breakdown:
1. Ask students to go to the website http://nlvm.usu.edu/en/nav/frames_asid_118_g_3_t_2.html or http://tinyurl.com/hanoi1

2. Ask students to click on the instructions button on the web page and use the instruction set to help describe the rules of the puzzle. The goal of the towers of Hanoi problem is to move a stack of disks from one peg to another in as few a moves as possible. You may move only one disk at a time. You may not stack larger disks on top of smaller disks.



3. Ask students to set number of disks = 3. Following instructions move all the discs on the left pole to the right pole. Ask students how many moves they made?



4. Ask students if they knew they should be able to complete the puzzle in 7 moves? A formula (2^n -1) is used to calculate the number of moves required for the optimal solution, where n is the number of disks. E.g. For 3 disks (2^n -1) = (2^3 – 1) = (8 – 1) = 7. Try solving the problem using 7 moves for 3 discs on 3 poles. To restart the game click "Reset". The PowerPoint slideshow that is used to accompany this lesson (Resource 1 Slide 4) contains a sample solution for 3 disks.
5. Tell students that the number of moves it takes to move all the disks from pole 1 to pole 3 is the interesting part of the problem. If you apply the rule (2^n -1) where n is the number of disks, how many moves would it take to solve the puzzle for 4 discs? Answer = (2^4 – 1) = (15 – 1) = 15
6. Ask students to set N to 4 and try this out.
7. Ask students to notice how the number of moves increases significantly as more disks are added.

Number of Disks	**Number of Moves Required to Solve the Problem**
6	63
9	511
15	32,767
25	33,554,431
40	1,099,511,627,775
64	9,223,372,036,854,775,808

8. Tell students that what they need to understand is that while you can solve this problem for 6 or 9 discs, it becomes very time consuming to solve for 64 discs. This does not mean that the problem cannot be solved for 64 disks. Computer science researchers are using other approaches **besides analysing all moves** to find a solution. How long would it take to solve the Towers of Hanoi for different numbers of discs? Let's say we have a computer that can execute a million instructions per second.

Number of Disks	**Time Taken to Solve the problem**
6	0.000063 seconds
9	0.000511 seconds
15	0.032767
25	33.55 seconds
40	11.14 days
64	292, 471 years

9. Finally, show students the implementation of the Towers of Hanoi in Scratch (Resource 2). Explain to students that they do not have to understand the scripts but that with more experience in Scratch they may be able to design their own implementation of the Towers of Hanoi in Scratch. When you open the project 'TowersofHanoi Scratch.sb' (Resource 2) click on the 'Show project notes' button to view instructions for the game (See Module 1, Lesson 2 for the location of the 'Show project notes' button in Scratch).

Resource 1

Towers of Hanoi

A PowerPoint slideshow guides the entire lesson

CD Resource

"M7L1R1 Towers of Hanoi.ppt"

Slide 1

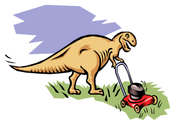
Solving Complex Problems

The Towers Of Hanoi

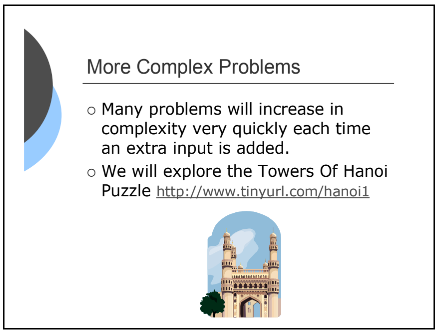
Slide 2

Linear Complexity

- Mowing the lawn is a problem that demonstrates linear complexity.
- If you double the size of the area that you want to mow, it will take twice as long to complete the task.

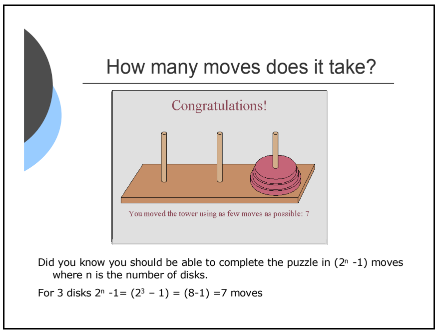


Slide 3



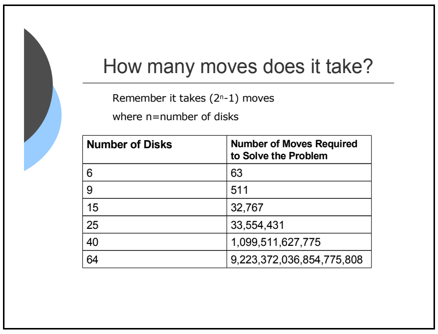
The puzzle was invented by the French mathematician Édouard Lucas in 1883. There is a legend about a Vietnamese or Indian temple which contains a large room with three time-worn posts surrounded by 64 golden disks. The priests of Brahma, acting out the command of an ancient prophecy, have been moving these disks, in accordance with the rules of the puzzle. According to the legend, when the last move of the puzzle is completed, the world will end. The puzzle is therefore also known as the Tower of Brahma puzzle. It is not clear whether Lucas invented this legend or was inspired by it. If the legend were true, and if the priests were able to move disks at a rate of one per second, using the smallest number of moves, it would take them roughly 272 thousand years to complete.

Slide 4



This slide shows how to complete the Tower of Hanoi puzzle for 3 disks using as few moves as possible.

Slide 5



How many moves does it take?

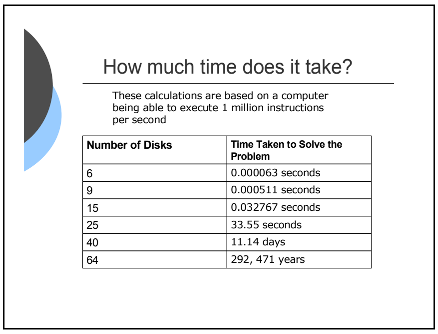
Remember it takes (2^n-1) moves

where n=number of disks

Number of Disks	Number of Moves Required to Solve the Problem
6	63
9	511
15	32,767
25	33,554,431
40	1,099,511,627,775
64	9,223,372,036,854,775,808

Notice how the number of moves increases significantly as more disks are added.
In mathematics the phrase exponential growth is used to describe the increase in the number of moves required to solve the problem in relation to the number of disks.

Slide 6



How much time does it take?

These calculations are based on a computer being able to execute 1 million instructions per second

Number of Disks	Time Taken to Solve the Problem
6	0.000063 seconds
9	0.000511 seconds
15	0.032767 seconds
25	33.55 seconds
40	11.14 days
64	292, 471 years

Executing 9,223,372,036,854,775,808 instructions would take 292,471 years to execute. People generally have a much shorter life expectancy than 292,471 years!

Resource 2

Towers of Hanoi in Scratch

An implementation of the Towers of Hanoi in Scratch

CD Resource

"M7L1R2 TowersofHanoi Scratch.sb"

Lesson 2 – The Travelling Salesman Problem

Resources:
Travelling Salesman (Resource 1)

Key Vocabulary:
Travelling Salesman Problem, Exponential Growth

Description:
Students will investigate how the amount of time required to execute an algorithm increases as the size of the input changes. Students will create solutions to the Travelling Salesman Problem for 10 cities. There are (n-1)! possible routes, where n is the number of cities. By following this pattern students will notice how the number of possible routes increases at an exponential rate as more cities are added to a journey.

Learning Objectives:

1. To investigate the increase in amount of time required to solve the Travelling Salesman Problem in relation to the increase in the number of cities in a journey.
2. To learn about applications based on the Travelling Salesman Problem.

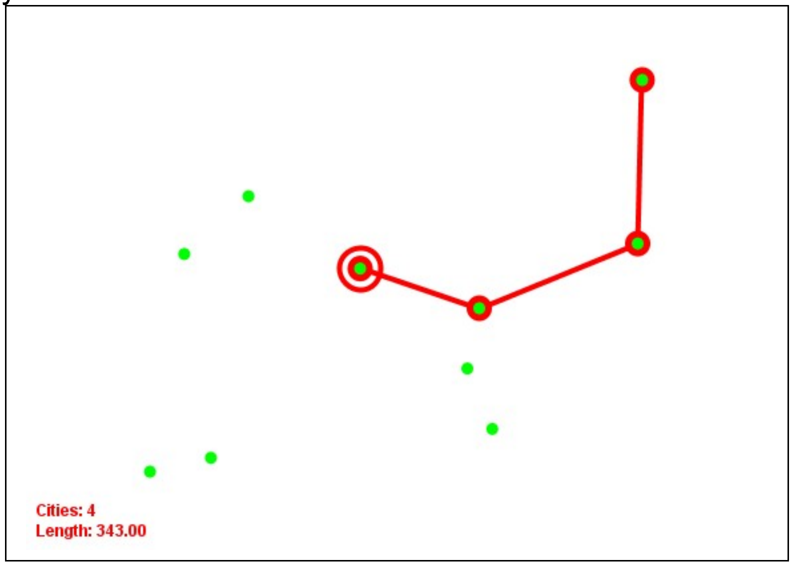
Lesson Introduction:

- In the previous lesson we used the Towers of Hanoi puzzle to show how a problem grew in complexity very quickly when we increased the number of disks.
- Open the PowerPoint Presentation for the lesson Travelling Salesman (Resource 1). Present slides 1 and 2.
- Tell students that today's lesson is based on the Travelling Salesman Problem. The rate of increase in complexity of this problem is even greater than that which we observed when investigating the Towers of Hanoi puzzle.

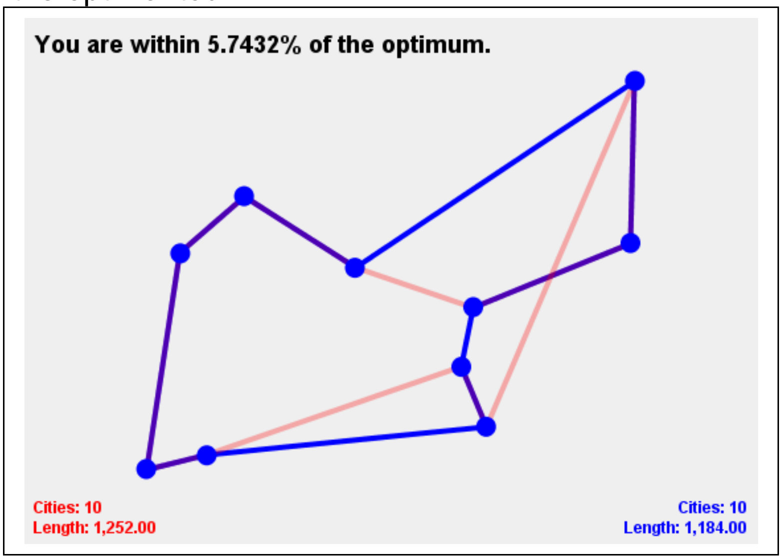
Lesson Breakdown:

1. The Travelling Salesman Problem involves finding the cheapest way of visiting a collection of cities and returning to the starting point knowing the cost of travel between each pair of cities.
Go to http://www.tsp.gatech.edu/games/tspOnePlayer.html or http://tinyurl.com/salesman1

2. You will be presented with a series of dots. These represent cities that you must visit.



3. You can click on the dots to start plotting your journey. All keyboard commands for this game will be activated/deactivated by clicking/un-clicking on the map with your mouse. Pressing the “x” key will remove the most recent link in your journey. Keep linking your cities until you have created a round trip to include links to every city on the map. Always remember that you are trying to take the shortest journey. If you manage to plot the shortest journey you will get a message saying you have the optimal tour.



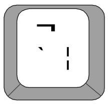
4. If you did not succeed in finding the optimal tour (shortest journey) you can press the “o” key and it should display the optimal tour. Try finding the shortest journey on two or three different maps. Press the spacebar to start on a new problem of the same size. Press + or -'to start on a bigger or smaller problem. An instruction set of commands appear on the webpage immediately underneath the game.

5. Work in pairs and try the 2 player version of the travelling salesman game. This is available at
http://www.tsp.gatech.edu/games/tspTwoPlayers.html
or http://tinyurl.com/salesman2

A knock out competition can be held to motivate students.

6. The keyboard commands for the 2 player game are activated by clicking on the map. The player using the blue circle steers using the arrow keys and places a marker using the enter key. The player using the red circle steers using the w,a,s,d keys and places a marker using the broken bar key (See below for an image of the broken bar key). An instruction set of commands appear on the webpage immediately underneath the game.

Broken Bar key



7. So what it this all about? As the number of cities increases, it becomes more and more difficult for a computer to check all possible tours. For example if you are visiting 4 cities, the number of possible routes you can take is (n-1)! = (4-1)! = 3! = (3x2x1) = 6 and you need to determine which of these 6 possible routes is the shortest.

8. By applying the (n-1)! rule it is easy to see that the number of possible routes increases significantly as more cities are added to the route.

Number of Cities	Number of Possible Routes
10	362, 880
12	39,916,800
15	87,178,291,200
25	620,448,401,733,239,439,360,000

9. Let’s say we have a computer that can execute a million instructions per second.

Number of Cities	Number of Possible Routes
10	0.36 seconds
12	39.91 seconds
15	24.21 hours
25	196 billion years

10. What you need to understand is that if you are trying to solve the problem by checking all possible tours you can solve this problem for 5, 7 or 10 cities, the problem cannot be solved in a reasonable amount of time for 25 cities. This does not mean that the problem cannot be solved for 25 cities. Computer science researchers have devised new approaches **besides analysing all possible routes** to solving the Travelling Salesman Problem. They use optimisation and approximation techniques to find a near-optimal solution.

Extension activity

- Google Maps can be used to plan the optimal 6 nations rugby trip between Dublin, Cardiff, Edinburgh, London, Paris and Rome. Use the website below and click the ‘Plan a Trip’ link to below to see it in action. The best flight distance should be close to 2600 miles
  http://www.tsp.gatech.edu/maps/index.html
  or http://tinyurl.com/salesman3

- Research the optimisation and approximation techniques used to solve the Travelling Salesman Problem or other similar problems.

Resource 1

Travelling Salesman

A PowerPoint slideshow guides the entire lesson

CD Resource

“M7L2R1 Travelling Salesman.ppt”

Slide 1

Solving Complex Problems

The Travelling Salesman Problem

Module 7- Solving Complex Problems

Slide 2

The Travelling Salesman Problem

- For a given set of cites, visit each city once and minimise the distance you travel.
- We will explore the puzzle http://www.tinyurl.com/salesman1

The Travelling Salesman Problem asks for an optimal tour through a specified set of cities. To solve a particular instance of the problem we must find the shortest tour and verify that no better tour exists.

Slide 3

How many possible routes?

Remember it takes (n-1)! moves

where n=number of cities

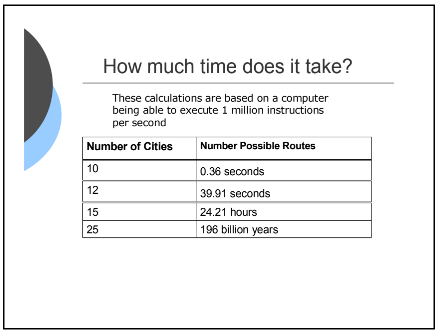
E.G. For 10 cities (n-1)! = (10-1)! = 9! = 9x8x7x6x5x4x3x2x1

Number of Cities	Number Possible Routes
10	362, 880
12	39,916,800
15	87,178,291,200
25	620,448,401,733,239,439,360,000

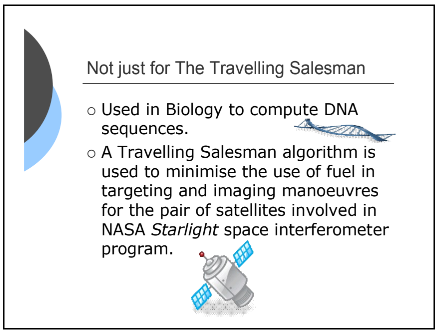
Notice how the number of possible routes increases significantly as more cities are added.

In mathematics the phrase exponential growth is used to describe the huge increase in number of possible routes in relation to the number of cities.

Slide 4



Slide 5



Although transportation applications are the most natural setting for the Travelling Salesman Problem, the model has led to many interesting applications in other areas.

References

Lesson 1

Towers of Hanoi Online Game

Utah State University National Library of Virtual Manipulatives
http://nlvm.usu.edu/en/nav/vlibrary.html

Resource 1

Slide 2

http://www.lawrencehallofscience.org/java/tower/towerhistory.html

Resource 2

http://scratch.mit.edu

Lesson 2

Travelling Salesman Website

Georgia institute of Technology Sub-site devoted to the Travelling Salesman Problem
http://www.tsp.gatech.edu/index.html

Resource 1

Slide 5

http://www.tsp.gatech.edu/apps/index.html